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4. Euclid's GCD Algorithm.
   A little tricky here!

Isoperimetry.

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Hypercube: $\Theta(1)$.

Surface Area is roughly at least the volume!

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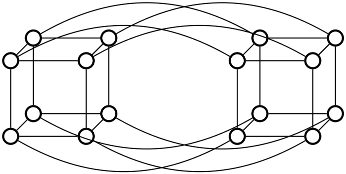

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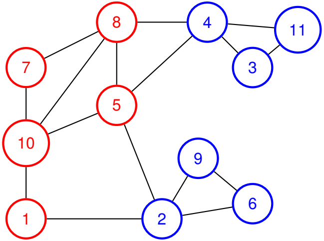
Restatement: for any cut in the hypercube, the number of cut edges is at least the size of the small side.

Cuts in graphs.



S is red, $V - S$ is blue.

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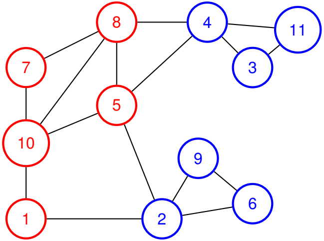

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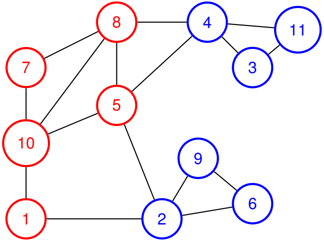

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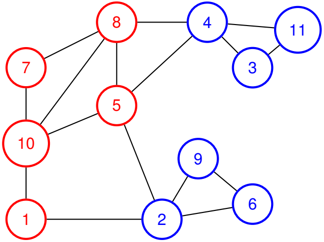

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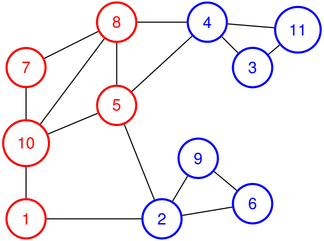

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What is size of cut?

Number of edges between red and blue. 4.

Hypercube: any cut that cuts off x nodes has $\geq x$ edges.

Proof of Large Cuts.

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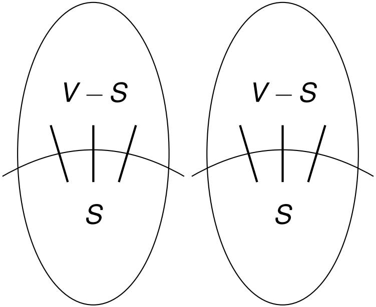

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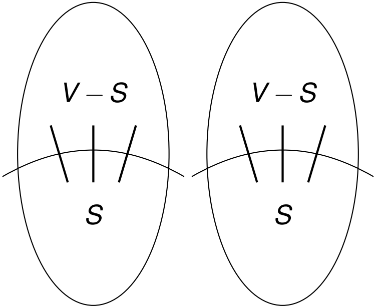

Case 2: Count inside and across.

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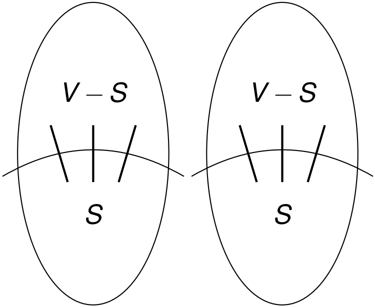

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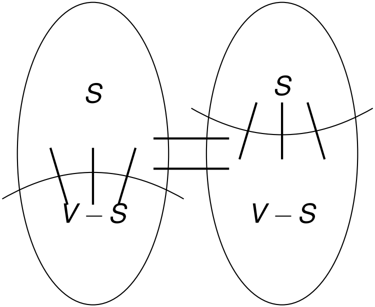

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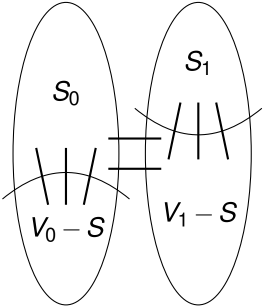
Proof: Induction Step. Case 2.

$$|S_0| \geq |V_0|/2.$$



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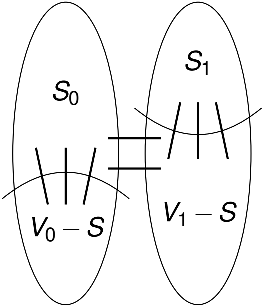

$|S_0| \geq |V_0|/2$.

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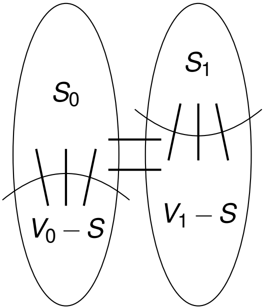

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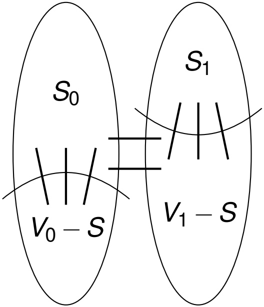

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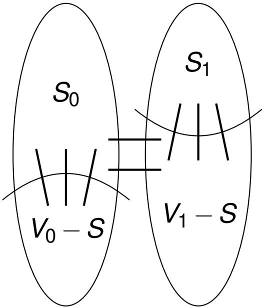

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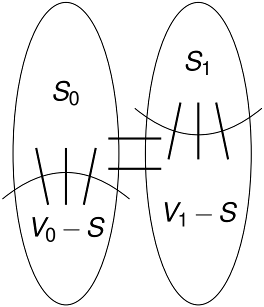

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Edges in E_x connect corresponding nodes.

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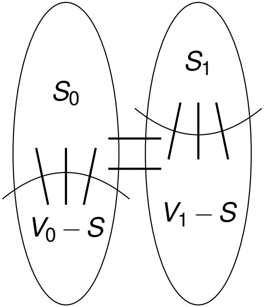

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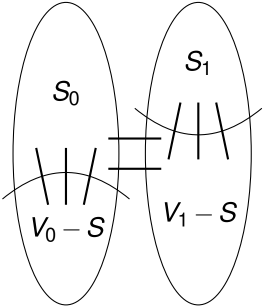

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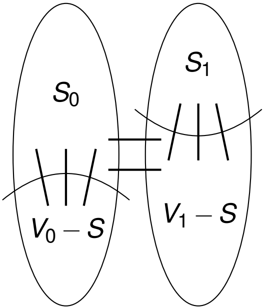

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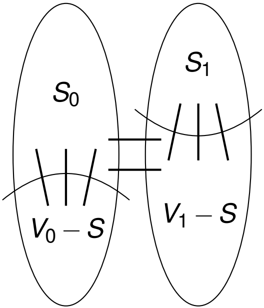

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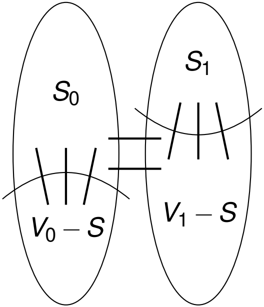

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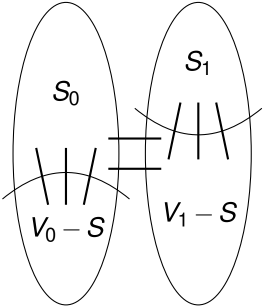

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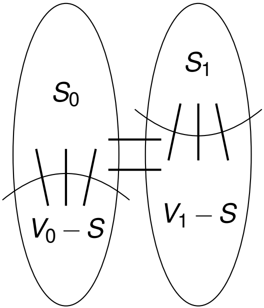

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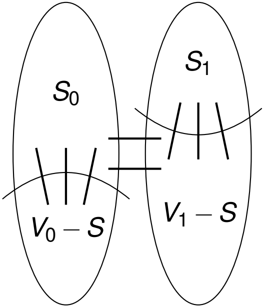

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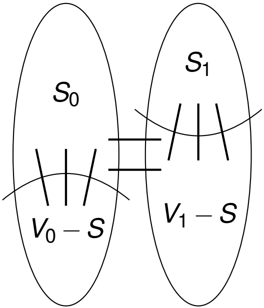

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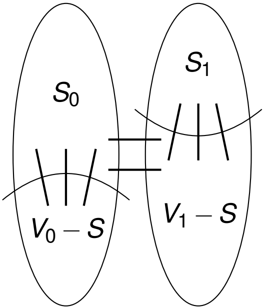

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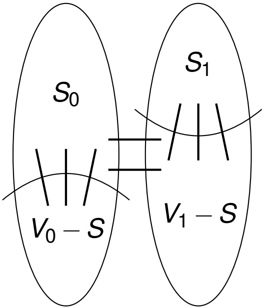

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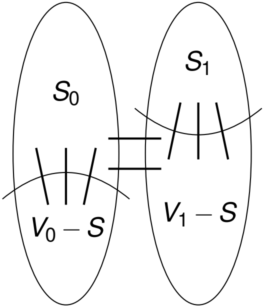

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$|V_0| = |V|/2 \geq |S|$. $\square$

Also, case 3 where $|S_1| \geq |V|/2$ is symmetric.

Hypercube proof: poll

Hypercube has large cuts proof uses these ideas:

(A) If cuts are same size on two sides it works by induction.
(B) Uses the fact that it is planar.
(C) Recursive definition of hypercube.
(D) If different size, can count edges between to subcubes.
(E) Applies Euler's formula.

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(A),(D), and (E).

Hypercubes and Boolean Functions.

The cuts in the hypercubes are exactly the transitions from 0 sets to 1 set on boolean functions on $\{0,1\}^n$.

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Central area of study in computer science!

Yes/No Computer Programs $\equiv$ Boolean function on $\{0,1\}^n$

Central object of study.

Modular Arithmetic.

Applications: cryptography, error correction.

Key ideas for modular arithmetic.

Theorem: If $d|x$ and $d|y$, then $d|(y-x)$.

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Proof:
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$\square$

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Proof:
$x = ad$, $y = bd$,
$(x-y) = (ad-bd) = d(a-b) \implies d|(x-y)$.

$\square$

Theorem: Every number $n \geq 2$ can be represented as a product of primes.

Proof: Either prime, or $n = a \times b$, and use strong induction. $\square$
(Uniqueness? Later.)

Poll

What did we use in our proofs of key ideas?

(A) Distributive Property of multiplication over addition.
(B) Euler's formula.
(C) The definition of a prime number.
(D) Euclid's Lemma.

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(A) and (C)

Next Up.

Modular Arithmetic.

Clock Math

If it is 1:00 now.

Clock Math

If it is 1:00 now.
What time is it in 2 hours?

Clock Math

If it is 1:00 now.
  What time is it in 2 hours? 3:00!

Clock Math

If it is 1:00 now.
- What time is it in 2 hours? 3:00!
- What time is it in 5 hours?

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours?

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

What time is it in 15 hours? 16:00!

Clock Math

If it is 1:00 now.

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Actually 4:00.

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If it is 1:00 now.

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Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours? 16:00!
Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.
Clock time equivalent up to to addition/subtraction of 12.

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

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Actually 4:00.

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Clock Math

If it is 1:00 now.

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Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours?

Clock Math

If it is 1:00 now.
- What time is it in 2 hours? 3:00!
- What time is it in 5 hours? 6:00!
- What time is it in 15 hours? 16:00!
  - Actually 4:00.

16 is the "same as 4" with respect to a 12 hour clock system.
- Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00!

Clock Math

If it is 1:00 now.
  What time is it in 2 hours? 3:00!
  What time is it in 5 hours? 6:00!
  What time is it in 15 hours? 16:00!
    Actually 4:00.

  16 is the “same as 4” with respect to a 12 hour clock system.
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What time is it in 100 hours? 101:00! or 5:00.

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
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Actually 4:00.

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What time is it in 100 hours? 101:00! or 5:00.
$101 = 12 \times 8 + 5$.

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If it is 1:00 now.
 What time is it in 2 hours? 3:00!
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16 is the “same as 4” with respect to a 12 hour clock system.
 Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00! or 5:00.
 $101 = 12 \times 8 + 5$.
 5 is the same as 101 for a 12 hour clock system.

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 What time is it in 2 hours? 3:00!
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What time is it in 100 hours? 101:00! or 5:00.
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 Clock time equivalent up to addition of any integer multiple of 12.

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Custom is only to use the representative in $\{12, 1, \ldots, 11\}$
 (Almost remainder, except for 12 and 0 are equivalent.)

Day of the week.

This is Thursday is February 11, 2021.

Day of the week.

This is Thursday is February 11, 2021.
What day is it a year from then?

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Number days.

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

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Day of the week.

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What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

Day of the week.

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What day is it a year from then? on February 11, 2022?

Number days.

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Today: day 4.

5 days from then.

Day of the week.

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What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from then. day 9

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from then. day 9 or day 2

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from then. day 9 or day 2 or Tuesday.

Day of the week.

This is Thursday is February 11, 2021.

What day is it a year from then? on February 11, 2022?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from then. day 9 or day 2 or Tuesday.

25 days from then.

Day of the week.

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25 days from then. day 29

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two days are equivalent up to addition/subtraction of multiple of 7.

Day of the week.

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    Number days.
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Today: day 4.
  5 days from then. day 9 or day 2 or Tuesday.
  25 days from then. day 29 or day 1. $29 = (7)4 + 1$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from then

Day of the week.

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25 days from then. day 29 or day 1. $29 = (7)4 + 1$

two days are equivalent up to addition/subtraction of multiple of 7.

11 days from then is day 1

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two days are equivalent up to addition/subtraction of multiple of 7.

11 days from then is day 1 which is Monday!

Day of the week.

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What day is it a year from then?

Day of the week.

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  25 days from then. day 29 or day 1. $29 = (7)4 + 1$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from then is day 1 which is Monday!

What day is it a year from then?
  Next year is not a leap year.

Day of the week.

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two days are equivalent up to addition/subtraction of multiple of 7.

11 days from then is day 1 which is Monday!

What day is it a year from then?

Next year is not a leap year. So 365 days from then.

Day of the week.

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- What day is it a year from then? on February 11, 2022?
  - Number days.
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Today: day 4.
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- 25 days from then. day 29 or day 1. $29 = (7)4 + 1$
  - two days are equivalent up to addition/subtraction of multiple of 7.
  - 11 days from then is day 1 which is Monday!

What day is it a year from then?
- Next year is not a leap year. So 365 days from then.
- Day 4+365 or day 369.

Day of the week.

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  What day is it a year from then? on February 11, 2022?
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Today: day 4.
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  25 days from then. day 29 or day 1. $29 = (7)4 + 1$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from then is day 1 which is Monday!

What day is it a year from then?
 Next year is not a leap year. So 365 days from then.
 Day 4+365 or day 369.
Smallest representation:

Day of the week.

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11 days from then is day 1 which is Monday!

What day is it a year from then?

Next year is not a leap year. So 365 days from then.

Day 4+365 or day 369.

Smallest representation:

subtract 7 until smaller than 7.

Day of the week.

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two days are equivalent up to addition/subtraction of multiple of 7.

11 days from then is day 1 which is Monday!

What day is it a year from then?

Next year is not a leap year. So 365 days from then.

Day 4+365 or day 369.

Smallest representation:

subtract 7 until smaller than 7.

divide and get remainder.

Day of the week.

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Number days.
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Today: day 4.
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25 days from then. day 29 or day 1. $29 = (7)4 + 1$
two days are equivalent up to addition/subtraction of multiple of 7.
11 days from then is day 1 which is Monday!

What day is it a year from then?
Next year is not a leap year. So 365 days from then.
Day 4+365 or day 369.
Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.
369/7

Day of the week.

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  What day is it a year from then? on February 11, 2022?
    Number days.
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  25 days from then. day 29 or day 1. $29 = (7)4 + 1$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from then is day 1 which is Monday!

What day is it a year from then?
  Next year is not a leap year. So 365 days from then.
  Day 4+365 or day 369.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.
  369/7 leaves quotient of 52 and remainder 5.

Day of the week.

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What day is it a year from then?

Next year is not a leap year. So 365 days from then.

Day 4+365 or day 369.

Smallest representation:

subtract 7 until smaller than 7.

divide and get remainder.

369/7 leaves quotient of 52 and remainder 5. $369 = 7(52) + 5$

Day of the week.

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11 days from then is day 1 which is Monday!

What day is it a year from then?
Next year is not a leap year. So 365 days from then.
Day 4+365 or day 369.
Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.
369/7 leaves quotient of 52 and remainder 5. $369 = 7(52) + 5$
or February 11, 2022 is a Friday.

Day of the week.

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  What day is it a year from then? on February 11, 2022?
    Number days.
      0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 4.
  5 days from then. day 9 or day 2 or Tuesday.
  25 days from then. day 29 or day 1. $29 = (7)4 + 1$
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    11 days from then is day 1 which is Monday!

What day is it a year from then?
  Next year is not a leap year. So 365 days from then.
  Day 4+365 or day 369.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.
  369/7 leaves quotient of 52 and remainder 5. $369 = 7(52) + 5$
    or February 11, 2022 is a Friday.

Years and years...

80 years?

Years and years...

80 years? 20 leap years.

Years and years...

80 years? 20 leap years. 366×20 days

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7?

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7?

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60$

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7?

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7$

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or February 11, 2101 is Saturday!

Years and years...

80 years? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 4.

Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Or February 11, 2101 is Saturday!

Further Simplify Calculation:

Years and years...

80 years? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 4.

Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Or February 11, 2101 is Saturday!

Further Simplify Calculation:

20 has remainder 6 when divided by 7.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or February 11, 2101 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.

Years and years...

80 years? 20 leap years. 366×20 days
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Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or February 11, 2101 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or February 11, 2101 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 6.

Years and years...

80 years? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or February 11, 2101 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 6. February 11, 2101 is Saturday.

Years and years...

80 years? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 4.

Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Or February 11, 2101 is Saturday!

Further Simplify Calculation:

20 has remainder 6 when divided by 7.

60 has remainder 4 when divided by 7.

Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.

Or Day 6. February 11, 2101 is Saturday.

"Reduce" at any time in calculation!

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.

Modular Arithmetic: refresher.

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if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
...or $x = y + km$ for some integer k.

Modular Arithmetic: refresher.

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Mod 7 equivalence classes:

Modular Arithmetic: refresher.

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...or x and y have the same remainder w.r.t. m.
...or $x = y + km$ for some integer k.

Mod 7 equivalence classes:
$\{\dots, -7, 0, 7, 14, \dots\}$

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
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Mod 7 equivalence classes:
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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
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...or $x = y + km$ for some integer k.

Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$

Modular Arithmetic: refresher.

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
...or $x = y + km$ for some integer k.

Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

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if and only if $(x - y)$ is divisible by m.
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Can calculate with representative in $\{0,\ldots,m-1\}$.

Notation

$x \pmod{m}$ or $\mod(x, m)$

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Work in this system.

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$6 \equiv 3 + 3 \equiv 3 + 10$

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Inverses and Factors.

Division: multiply by multiplicative inverse.

$$2x = 3 \implies (\frac{1}{2}) \cdot 2x = (\frac{1}{2}) \cdot 3 \implies x = \frac{3}{2}.$$

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Check!

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$\quad 8x = 10 \pmod{7}$
$\quad x = 3 \pmod{7}$
Check! $4(3) = 12 = 5 \pmod{7}$.

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For 8 modulo 12: no multiplicative inverse!

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"Common factor of 4"

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"Common factor of 4" $\implies$
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"Common factor of 4" $\implies$
$8k - 12\ell$ is a multiple of four for any ℓ and k $\implies$
$8k \not\equiv 1 \pmod{12}$ for any k.

Poll

Mark true statements.
(A) Mutliplicative inverse of 2 mod 5 is 3 mod 5.
(B) The multiplicative inverse of $((n-1) \pmod{n}) = ((n-1) \pmod{n}))$.
(C) Multiplicative inverse of 2 mod 5 is 0.5.
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(E) $(-1)x(-1) = 1$. Woohoo.
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Poll

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(F) Multiplicative inverse of 4 mod 5 is 4 mod 5.

(C) is false. 0.5 has no meaning in arithmetic modulo 5.

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Very different for elements with inverses.

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Not a bijection.

Poll

Which is bijection?

(A) $f(x) = x$ for domain and range being $\mathbb{R}$

(B) $f(x) = axmod(n)$ for $x \in \{0, ..., n-1\}$ and $gcd(a, n) = 2$

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(B) is not.

Only if

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Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.
Equal to 1? Mutliplicative inverse.

Algorithm: Try all numbers up to x to see if it divides both x and m.

Very slow.

Inverses

Next up.

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Euclid's Algorithm.

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Proof...: Similar.

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GCD Mod Corollary: $\gcd(x,y) = \gcd(y, \mod(x,y))$.

Hey, what's $\gcd(7,0)$? 7 since 7 divides 7 and 7 divides 0
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Theorem: (euclid x y) $= \gcd(x,y)$ if $x \geq y$.

Proof: Use Strong Induction.
Base Case: $y = 0$, "x divides y and x"
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