1 Number Game

Sinho and Vrettos are playing a game where they each choose an integer uniformly at random from $[0, 100]$, then whoever has the larger number wins (in the event of a tie, they replay). However, Vrettos doesn't like losing, so he's rigged his random number generator such that it instead picks randomly from the integers between Sinho's number and 100. Let S be Sinho's number and V be Vrettos' number.

(a) What is $\mathbb{E}[S]$?

(b) What is $\mathbb{E}[V|S = s]$, where s is any constant such that $0 \le s \le 100$?

(c) What is $\mathbb{E}[V]$?

2 Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation.

(a) If we roll a die until we see a 6, how many ones should we expect to see? Notice that this is the MMSE of the number of ones we see given that we've seen a 6.

(b) If we roll a die until we see a number greater than 3, how many ones should we expect to see?

3 Marbles in a Bag

We have r red marbles, b blue marbles, and g green marbles in the same bag. If we sample marbles with replacement until we get 3 red marbles (not necessarily consecutively), how many blue marbles should we expect to see? (*Hint*: It might be useful to use Law of Total Expectation, $E(Y) = E(E(Y|X))$)