CS 70 Discrete Mathematics and Probability Theory
Spring 2021

DIS 0A

1 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \land (Q \lor P) \equiv P \land Q$

(b) $(P \lor Q) \land R \equiv (P \land R) \lor (Q \land R)$

(c) $(P \land Q) \lor R \equiv (P \lor R) \land (Q \lor R)$

2 XOR

The truth table of XOR (denoted by $\oplus$) is as follows.

A	B	A $\oplus$ B
F	F	F
F	T	T
T	F	T
T	T	F

1. Express XOR using only $(\land, \lor, \neg)$ and parentheses.

2. Does $(A \oplus B)$ imply $(A \lor B)$? Explain briefly.

3. Does $(A \lor B)$ imply $(A \oplus B)$? Explain briefly.

3 Converse and Contrapositive

Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

4 Equivalences with Quantifiers

Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers.

(a)	$\forall x \left((\exists y\, Q(x,y)) \Rightarrow P(x) \right)$	$\forall x\, \exists y \left(Q(x,y) \Rightarrow P(x) \right)$
(b)	$\neg \exists x\, \forall y \left(P(x,y) \Rightarrow \neg Q(x,y) \right)$	$\forall x \left((\exists y\, P(x,y)) \wedge (\exists y\, Q(x,y)) \right)$
(c)	$\forall x\, \exists y \left(P(x) \Rightarrow Q(x,y) \right)$	$\forall x \left(P(x) \Rightarrow (\exists y\, Q(x,y)) \right)$